Why do we do proofs?

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1. Aren't all mathematical statements either clearly true or clearly false, after checking a few examples and/or doing some calculations?
2. Why should we prove anything at all?
3. How can we be absolutely sure whether a statement is true or false?
4. Why should we prove statements which appear to be intuitively obvious?
5. Why are definitions of concepts important?
6. Do we need to memorize lots of proofs?

Problem 1

For which prime numbers p, if any, is $p + 1$ a perfect square?

You should find one example quite quickly. Are there any others? Are you sure? Can you generalize this result?

Problem 2 (Fermat's Last Theorem!)

Prove that, whenever x, y, z and n are positive integers with $n > 2$, then

$$x^n + y^n \neq z^n .$$

That's rather a tricky one. It took over 350 years to find a proof!

p prime so that $p+1$ is a perfect square?

Example $p = 3$, $p+1 = 4 = 2^2$.

Claim: there are no others

Proof. Suppose that p is prime and $p+1$ is a perfect square. Say $p+1 = n^2$ where n is a non-negative integer.

Clearly n must be >1 here.

Then $p = n^2 - 1 = (n-1)(n+1)$.

So p can not be prime unless $n-1=1$, i.e. $n=2$, $n^2-1 = 3 = p$.

So 3 is the <u>only</u> solution. $\square$

Two hospitals (Hospital A and Hospital B) each claim to be better at treating a certain disease than the other.

Hospital A points out that it cured a greater percentage of its male patients last year than Hospital B did, and that it also cured a greater percentage of its female patients last year than Hospital B did.

However, Hospital B points out that, overall, it cured a greater percentage of its patients last year than Hospital A did.

Problem 3 (Serious answers please!)

Given that none of the numbers involved are zero, is it possible that both hospitals have got their calculations right?
If so, which hospital would you rather be treated by?

EXAMPLE

	M	F	Total
A	50/100 50%	1/1 100%	51/101 50.5%
B	24/50 48%	49/50 98%	73/100 73%

This well-known phenomenon is called Simpson's paradox.

If you are unclear about the precise definitions of the concepts you are working with, it is very hard to be certain about the answers to questions concerning these concepts.

Often the answers depend on the definitions you are using.

Here are some questions for you to think about which might illustrate this (you may not all agree about the answers!)

Problem 4

Is every square a rectangle?

Problem 5

Is every oblong a rectangle?

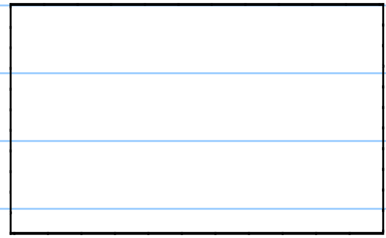
Problem 6

Is every rectangle an oblong?

Square –

is this a rectangle?

An oblong?



Rectangle –

is this an oblong?

Are all oblongs like this?